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Methodological Aspects of the Theory of Objectification

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Abstract

In this article, I focus on the methodology of a specific theory of teaching and learning: the theory of objectification. Inspired by dialectical materialism and Vygotsky's psychology, the theory of objectification posits the goal of Mathematics Education as a dynamic political, societal, historical, and cultural endeavour aimed at the dialectical creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted and always evolving mathematical discourses and practices. In the first part, I briefly sketch the general lines of the theory. In the second part, emphasizing the semiotic and embodied nature of teaching and learning, I discuss the methodology of the theory, stressing in particular its task design, data collection, and data analysis components.

Keywords: Theory of objectification. Methodology. Consciousness. Semiotics. Activity.

Introduction

There seems to be a consensus around the claim that it is impossible to carry out an investigation on learning without a clear methodology. What is less obvious is to reach a consensus on what a methodology is and how it functions within a theory of learning. In a general sense, a methodology can be considered as a kind of *method*, that is, a procedure to follow. This is what an etymological analysis suggests. Method comes from the Greek *methodos*, a word made up of *meta* —"after"—and *hodos* —"a traveling"—meaning hence "a following after" (Harper, 2013). However, things get quickly complicated by the fact that, to make a methodology operational, we need to specify the "things" (i.e., the objects of inquiry) that we are after. Is it some *evidence* that learning has occurred or is occurring? What conditions does a methodology have to fulfil in order to provide us with *convincing* evidence?

There is no straightforward answer to this question. The Ancient Greeks, for instance, favoured a kind of contemplative process. They considered a method to be something to help us make sense of things *already out there*, by looking at them attentively. Classifications, like

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the botanical ones carried out by Aristotle, were the tools with which to do that. Finding the *genus* and its variants was the method used to ascertain the limits of the *species*. But, in this line of thought, the observed objects were not *forced* to appear. They were there, accessible to be collected and inspected. We have to wait until the late Middle Ages and early Renaissance to find the idea that we can *force* the object under scrutiny to appear. That was the role of the *scientific experiment*.

The idea of the scientific experiment led to a reconceptualization of the objects of investigation and led us to reflect on what was meant by a "fact" and how a fact was evident or constituted evidence of something more general. For the Ancient Greeks, facts were subjected to universal propositions. In an important sense, a fact illustrates a general principle. In *Posterior Analytics*, Aristotle claims that sense perception must be concerned with particulars, whereas knowledge depends upon the recognition of the universal (Aristotle, *Posterior Analytics*, Book 1, Part 31). Hence, for Aristotle and the Ancient thinkers, a fact embodies something that transcends it. By contrast, since the early 17th century, under the influence of Francis Bacon, facts were understood by some natural philosophers as theory-free particulars. As Mary Poovey (1998, p. xviii) notes in her book *A History of the Modern Fact*, some scientists argued that "one could gather data that were completely free of any theoretical component". With Francis Bacon (1906) particulars gained an epistemological prestige. Facts —the good ones, those that are really convincing—could be displayed as tomatoes on a table. They speak for themselves.

The previous comments underline the idea that a methodology does not operate independently of certain assumptions about the "nature" of facts. In Aristotle's approach, the fact refers to general principles; the fact is a particularisation of the general. In the Baconian approach, the fact generates the principle through an inductive process. In other words, a methodology, M, is always in a relationship with some theoretical principles, P. These theoretical principles do not only provide the conceptual support to conceive of facts and to endow the methodology with its convincing epistemological dimension, but they also allow us to formulate in specific ways the research questions. Indeed, a research question requires not only a language to be expressed, but, overall, theoretical principles to make sense. This is why it is so difficult to write grant proposals The funding agencies require us to state, right away, the research questions, as if they would make sense by themselves, like Baconian facts.

From the previous considerations we can see that a methodology, M, can make sense only through its interrelationship with a set of theoretical principles, P, and the research questions, Q, that we seek to answer. Elsewhere (Radford, 2008a), referring to the research field of Mathematics Education, I have termed *theory* the triplet (P, M, Q). The arrows in Figure 1 indicate that there is a dialectical relationship between them. As a result, genetically speaking, P is not prior to M or to Q. In fact, in their interrelationship, each one of them alters the others.

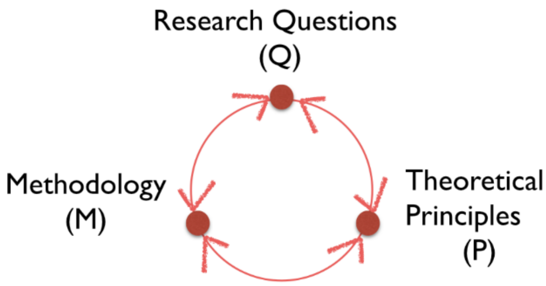


Figure 1- A theory made up of three components (P, M, Q).

In this article, I focus on the methodology of a specific theory of teaching and learning: the theory of objectification (2008b, 2012a, 2014a). In the first part, I briefly sketch the general lines of the theory. In the second part I focus on the methodological component.

A sketch of the theory of objectification

To a large extent, mathematics education has been defined (implicitly or explicitly) as the diffusion of mathematical knowledge (see, e.g., Brousseau's (1997) theory of didactical situations) or the personal growth of autonomy and cognitive structures (see, e.g., Cobb's (1988) socio-constructivism). The theory of objectification—a theory of teaching and learning—follows a different path. This theory conceives of mathematics education as embedded in a larger educational, transformational project. It posits the goal of Mathematics Education as a dynamic political, societal, historical, and cultural endeavour aimed at the dialectical creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted and always evolving mathematical discourses and practices. This general idea of education serves to redefine the notions of the teachers and the students, the processes of teaching and learning, and brings forward a dialectical materialist conception of knowledge.

Let me start by referring to the concept of knowledge.

Knowledge

The theory of objectification draws on dialectical materialism and its idea that knowledge is not something that individuals possess, acquire, or construct through personal actions. In dialectical materialism, knowledge is not a psychological or mental entity. The dialectical materialist idea of knowledge rests on the distinction between the *Potential* (something that may happen, i.e., possibility) and the *Actual* (its happening). The objects of knowledge are on the side of the Potential: a system of social-historical-cultural entities, or as Evald Ilyenkov (2012, v. 20, n. 2, p. 150) put it a, "complete totality of possible interpretations—those already known, and those yet to be invented". Knowledge includes possibilities of making calculations, or thinking and classifying spatial forms in certain "geometric" manners; possibilities of taking courses of action or imagining new ways of doing things, etc. This is what school knowledge is when the student enters the school for the first time—*pure open possibility*.

To assert that knowledge is possibility does not amount to saying that knowledge is something eternal, static, or independent of all human experience (as in Kant's concept of *things-in-themselves* or as in Plato's *forms*). In fact, knowledge results from, and is produced through, human social labour. Knowledge is a *cultural dialectic sensuous synthesis* of people's doings. More precisely, knowledge is a dynamic and evolving implicit or explicit culturally codified way of doing, thinking, and relating to others and the world.

Knowledge as possibility means that knowledge is *not immediate*. In order for it to become an object of thought and consciousness, knowledge has to be set into *motion*. That is to say, it has to acquire cultural determinations. We can use a metaphor and think of knowledge as an archetype of actions, interpretations, reflections, etc.—a system of formal configurations. These configurations are *abstract*, that is to say, they lack content (or cultural determinations). These configurations cannot be the object of senses, consciousness, and thought. The only manner by which knowledge can acquire content and cultural determinations is through specific *activities*. Let us take the example of algebraic knowledge. Algebraic knowledge is not the sequence of marks we see on a paper. These marks are signs, or traces of human activity. Algebraic knowledge is pure possibility—possibilities of thinking about indeterminate and known numbers in manners that are opened up by certain historically constituted analytical ways of thinking. Algebraic knowledge can become an object of thought and interpretation only by being put into motion and being made into an object of senses and consciousness through sensuous and sign-mediated specific problem-solving and problem-posing activities.

In more general terms, knowledge moves through activity from an indeterminate form of possibilities to a determinate singularized form filled with content or concrete determinations (e.g., the singularized knowledge-form that results from the students dealing with some *specific* equations, like $2x + 3 = x + 6$). Figure 2 illustrates these ideas diagrammatically. The abstract potential is put into motion through activity and is actualized in a singular.

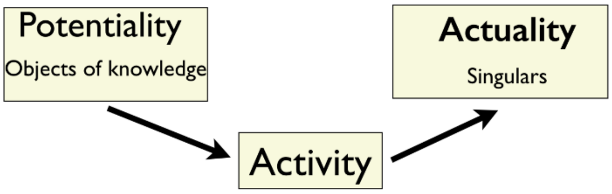


Figure 2- Activity as that which puts potentiality into motion and actualizes or concretizes it in a singular.

Learning

From the previous conception of knowledge we can refer to learning as follows. Learning can be theorized as those processes through which students gradually become acquainted with historically constituted cultural meanings and forms of reasoning and action. Those processes are termed processes of *objectification* (Radford, 2002). They entail a moment of *poēsis*: a moment of "bringing-forth" something to the realm of attention and understanding. *Poēsis* is a creative moment of disclosure–the event of the thing in consciousness (Radford, 2010).

Let me comment on four important elements of objectification as defined above.

First: objectification is a *process*—an unfinished and unending process. We can always learn more. This is why, in our recent work, we do not say that objectification has occurred—we do not say that student A objectified X. Rather, we talk about objectification as an unfolding phenomenon. We talk about students in the process of objectifying something instead of having objectified something.

Second: objectification is a *social* process—that is, a process that we carry out *with others*, whether or not the others are there, face to face, or remotely, virtually, or through language, or artifacts (books or other mediating cultural elements).

Third: that in the course of a process of objectification, the students become *conscious* or *aware* of something that was already there (e.g., how to think about, and solve, linear equations). What the third point stresses is that learning is defined as a *problem of consciousness*. There are many theories of learning that do not need to refer to consciousness.

However, if we remove the construct of consciousness from the theory of objectification, there is no longer theory: it collapses. Naturally, we will need to specify what we mean by consciousness. I shall return to this point later when I address the question of what counts as evidence in the methodology. For the time being suffice it to say that consciousness is not the metaphysical construct of idealism—something buried in the depths of the human soul. As we shall see, consciousness, from the dialectical materialist perspective adopted here, is a concrete theoretical construct that is open to empirical investigation, for as Voloshinov (1973, p.11) put it, "consciousness . . . is filled with signs. Consciousness becomes consciousness only once it has been filled with . . . (semiotic) content, consequently, only in the process of social interaction".

Fourth: to state that, in a process of objectification, the students become progressively acquainted with historically constituted cultural meanings and forms of reasoning and action, does not amount to stating that the students have to agree with those forms of reasoning and action. Here, to become acquainted means *understanding*—a socially responsible and conceptually articulated understanding of something even if we do not agree with it. This point is very important to bear in mind, because without it, we may fall into the traps of a conformist and reproductive pedagogy. The theory of objectification is a dialectical materialist theory based on the idea of Otherness or alterity. Learning is to encounter something that is *not me*. If in the course of my deeds I come to understand only that which I have produced, then I have not learned anything. I am the identity of myself and my deeds. I am the identity of subject and object. The theory of objectification posits the subject and the object as *heterogeneous* entities. In encountering the cultural object, that is to say, an object of history and culture, it *objects* me. Etymologically speaking, it means that I *feel* it as something *alien* and, in the poetic encounter of the object and myself, I come to cognize it, not only cognitively but emotionally, sensuously, even if I do not agree with it. This encounter of the object is what objectification is about (and from where the theory takes its name). But because of its cultural nature, the object that I encounter is not merely an object, but a cultural object, so that what I encounter in the end is not only the object but also the *Other in the object*. Thus, when a baby stretches her arm to grasp an object, what she grasps is not the object *as* object, the thing in its materiality, as in Piaget's account, but also the human activity that has produced the object. Marx notes: "Even when sensible reality is reduced to a stick, au strict minimum . . . it implies the activity that has produced the stick" (quoted in HENRY, 1976, p. 361).

Hence, objectification is more than the connection of the two classical epistemological poles, subject and object: it is in fact a dialectical process—that is, a transformative and creative process between these two poles that mutually affect each other. As a result, in the course of learning, the subject comes into contact with cultural knowledge and, in so doing, on the one hand affects the cultural knowledge through the always new event of its actualization (see Figure 2), and, on the other hand, comes to cognize and recognize herself in a reflective process that we call *subjectification*. Subjectification is the making of the subject, the creation of a particular (and unique) subjectivity that is made possible by the activity in which objectification takes place. This is why mathematics classrooms do not produce only knowledge, but subjectivities too, and that learning is both a process of knowing and a process of becoming (Radford, 2008b).

What are the kinds of research questions that the theory of objectification asks? The theory of objectification is an attempt to understand learning not as the result of the individual student's deeds (as in individualist accounts of learning) but as a cultural-historical situated process, and to offer accounts of the entailed processes of knowing and becoming. It seeks to study the manners by which the students become progressively aware of historically and culturally constituted forms of thinking and acting, and how, as subjectivities in the making, teachers and students position themselves in mathematical practices.

We are ready now to turn to the methodology.

Methodology

The unit of analysis

Figure 2 puts into evidence the classroom activity's role in the actualization of knowledge. What students encounter is not knowledge as potentiality, but knowledge as actualized through activity. Mathematics is much like music in this respect. What we encounter when we hear Beethoven's Fifth Symphony is not Beethoven's Fifth as potential but always as an actualization of it by a specific orchestra at a certain moment. What makes the appearance of music/mathematical knowledge possible is the activity that actualizes it. In the theory of objectification activity is indeed taken as the methodological *unit of analysis*.

Activity in the theory of objectification does not merely mean to do something. Activity (*Tätigkeit* in German and *deyatel'nost'* in Russian) refers to a system that contributes to the satisfaction of collective needs and that operates within a specific division of labour. It is in this sense that activity appears as the minimal unit that reproduces society as a whole. It rests on a specific conception of individuals as natural beings of needs (Radford, forthcoming).

The design of classroom activities

Knowing

Activities in general and classroom activities in particular are characterized by their *object* (Leont'ev, 2009). The object of a mathematics classroom activity may be, for example, the objectification (i.e., encounter) of algebraic forms of thinking about linear equations. Naturally, in general, the teacher and the students have a different grasp of this object. The object, which has a didactic intention, is not necessarily clear to the students from the outset. The object of the activity will be revealed to the students as they engage in the classroom activity.

Now, in general, for the classroom activity to move towards its object, it is often pedagogically necessary to introduce some *goals*. These goals can be, if we continue with our algebra example, to algebraically solve problems about linear equations. To reach the goals of the activity, specific *tasks* have in turn to be envisioned. They may appear as a sequence of related problems of increasing conceptual difficulty.

The teachers of the classrooms that we investigate participate in the design of the activity and play an important role in the selection of problems and their conceptual organization. We discuss which problems to include, when, and why. In the course of many years of conducting research with teachers and their students we have developed a basic working list that helps us go through the design of the activity. The problems we pose:

a) Take into consideration what the students know;

b) Are interesting from the students' point of view;

c) Open up a space of critical reflection and interaction through small groups discussion, between small groups discussions, and general discussions;

d) Make meaningful the target mathematical concepts at deep conceptual levels;

e) Offer the students the occasion to reflect mathematically in different ways (not only through the lenses of dominant mathematics); and

f) Are organized in such a way that there is a conceptual thread oriented towards problems of increasing mathematical complexity. (Radford, Demers, & Miranda, 2009)[2]

The *object—goal—task* structure is hence a central part of the *design of the classroom activity*. It corresponds to the left arrow in Figure 2. Let me call it the ϕ arrow. This arrow refers to the pedagogical intention of the classroom activity. It involves an epistemological analysis of the target mathematical content and classroom interaction that we complement with an a priori analysis (Artigue, 1988, 2009). The a priori analysis is a reflection of how things might occur in the classroom. Teachers and researchers may have an idea, but the process of the actualization of knowledge is not a mechanical one. It will depend on how students and teachers engage in the activity, how they respond to each other, etc. How things actually turn out in the classroom is what the right arrow in Figure 2 means. Let me call this arrow the Θ arrow. We have then Figure 3.

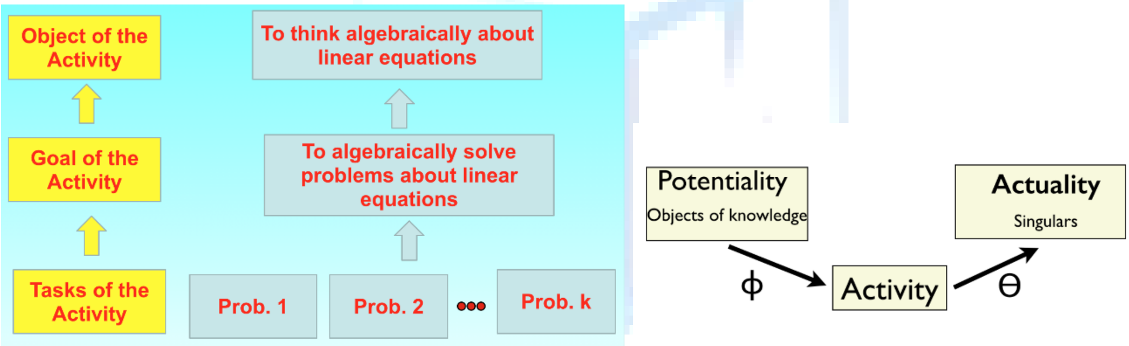


Figure 3- Left, the object-goal-task structure; right, the design (ϕ arrow) and implementation of the activity (Θ arrow).

The Θ arrow indicates the specific actualization of knowledge, as produced by the teacher and students' classroom activity. The specific actualization of knowledge is better understood as an emergent process. The adjective "emergent" means that the classroom is envisioned as a *system* that evolves through "states" and that this evolution cannot be determined in advance. The classes we work with are usually divided into small groups of two to three or four students. The first state of the emerging system, identified by the Θ arrow, is a presentation of the activity by the teacher (see Figure 4). Then, the students are invited to work in small groups (see "Small group work" in Figure 4). Then, the teacher visits the various groups

[2] Several examples of task design can be found in Radford and Demers (2004) and Radford, Demers, and Miranda (2009)—these are books intended mainly for teachers.

and asks questions to the students, gives feedback, etc. (see “Teacher-students discussion” in Figure 4). At a certain point, the teacher may invite the class to a general discussion where the groups can present their ideas and other groups can challenge them, suggest something else, or improve and generalize what other groups have produced (see “General discussion” in Figure 4). The lesson may end there or continue with additional small group discussion, etc.

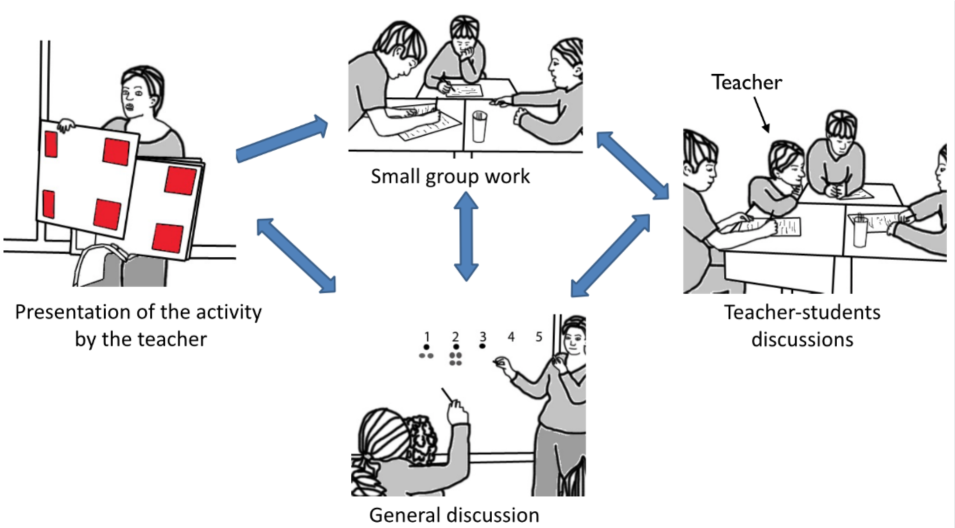


Figure 4 - Classroom activity as an emerging system.

Becoming

Hence, the ϕ and Θ arrows refer respectively to the pedagogical planning of the activity and its real actualization. They rest on two basic ideas of all classroom activity, namely (1) the forms of classroom knowledge production and (2) the forms of human collaboration. While the former refers to notions about knowledge, truth, and forms of inquiry and proving that are promoted in the classroom, the latter refers to the modes of classroom interaction that are fostered in the classroom and the underlying ethical dimension of the student-student and teacher-student relations.

The forms of classroom knowledge production and the forms of human collaboration that we nurture in the classrooms we work with are based on an ethic that fosters modes of collaboration of a non-utilitarian and non self-centred nature—modes of human collaboration and interaction that promote critical stance, solidarity, responsibility, and the care of the other. This communitarian ethic envelops the object-goal-task structure of the activity (see Figure 5), shaping the manner in which students engage with others in discussions, debates, and controversies, thereby realizing culturally evolved critical forms of subjectivity. The communitarian ethic is fundamental to the students' processes of subjectification and becoming.

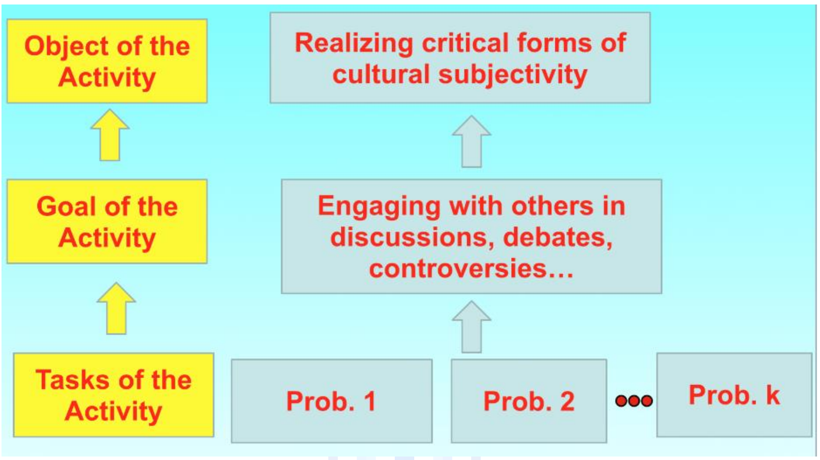


Figure 5 - The communitarian ethic in the object-goal-task activity structure.

Naturally, the chief elements of the communitarian ethics we bring forward are not natural. They are the result of cultural evolution. Here, the teacher plays a fundamental role in promoting them in the classroom (see, e.g. Radford, 2012b, 2014b).

Data collection and analysis

In the previous section we dealt with the design and implementation of the classroom activity. In this section we deal with the data collection and analysis. Our data collection comes from classroom lessons that are part of the regular school mathematics program. The period of data collection varies. Usually we follow the same class for three to five or six years (although we have also worked with classes for only one year in specific projects with particular school boards in Ontario). We follow the class for several weeks a year, not necessarily consecutive weeks. Data is collected through a four-phase process:

1) Video- and audio-recording:

We use three to five video cameras, each filming one small group of students. The cameras are equipped with long-life batteries to avoid unnecessary cables in the classroom. Since our classrooms are very noisy as a result of the students' interaction, we attach an external stereo microphone to each camera. In addition to this, we place a voice recorder on the desk of the student group being videotaped. The voice recorder has two functions: it can be used as backup in case something goes wrong with the camera's external microphone; it can also help us hear what some students say, in case they talk low or are far away from the external microphone. A voice recorder is also put beside the Smart or White Board to record the teacher's and student's voices when they come to the front of the classroom.

2) Student activity sheet:

Each student receives a student activity sheet where they keep track of their ideas and write their answers to the problems (task of the activity). We provide the students with black ink pens (not pencils) and encourage them not to strike through discarded ideas so that whatever they write can be still be read, allowing us to follow their line of thought. If the activity is not finished during the mathematics lesson (which is usually the case), we collect their activity sheets, photocopy them in black and white, and come back the next day with the photocopies. We give them the photocopies and blue ink pens. The use of a different ink colour allows us to distinguish between what the students will write during the mathematics lesson and what they wrote in the previous lesson.

3) Smart board documents:

Usually, the schools are equipped with Smart Boards (SB). We keep a digital copy of everything that was written on the SB. The SB documents contain some teacher explanations and traces of what students present during the general discussion phases.

4) Field notes:

The teacher and the members of the research team who videotape the small group work use voice recorders to create field notes after each mathematics lesson. These field notes contain remarks about what happened in the classroom; for example, notes about the ethical dimensions (collaboration, responsibility, solidarity, etc.) or mathematical understandings that are considered interesting for further investigation.

Data processing and storage

The student activity sheets are scanned and, along with the SB documents and field notes, they are transferred to a dedicated server with the videos and their transcriptions. They are stored in accordance with the structure shown in Figure 6, based on the case of a three-year longitudinal research project. The members of the research team have online access to the secured server.

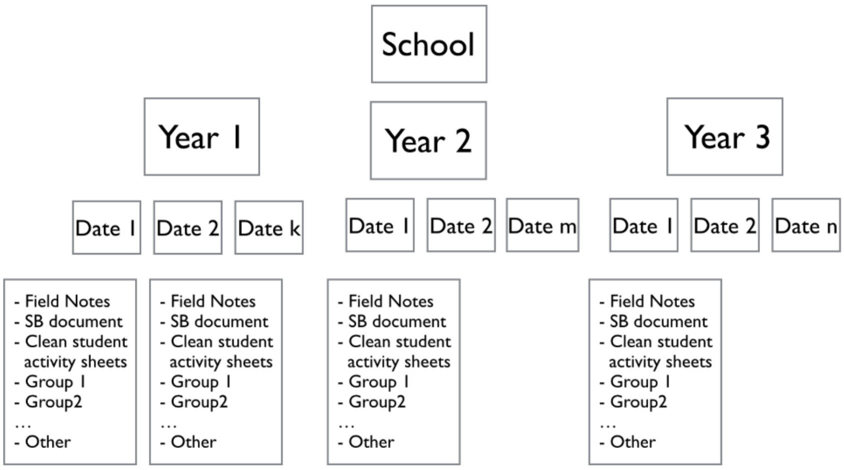


Figure 6 - Structure of the data storage in a dedicated server.

For lack of space, Figure 6 shows only subfolders for Dates 1 and 2 for Year 1. For Year 2 and 3 Figure 6 shows the subfolder for Date 1 only. As we can see, the fourth level contains data for each group (Group 1, Group 2, … Group j, where j = number of videotaped groups = number of video cameras used). Each Group k folder ($1 \leq k \leq j$) contains the video, the video transcription, the student activity sheets for each student of the group, as well as pictures and sketches generated through the contextual transcription analysis (see below). The Other folder contains scanned student activity sheets from those students who were in groups that we did not videotape and who nonetheless returned the ethic consent form signed by a parent/guardian and the student, as approved by the university and the corresponding school board.

If teachers give individual written assessments to the students, the student assessment sheet is scanned and added to the data corpus as well as a field note document that contains the teacher's remarks about the assessment.

The number of teachers we work with in a year may vary from one teacher up to a dozen teachers.

Data analysis

What do we do with the data? We seek to document the processes of objectification and subjectification. As mentioned previously, by objectification we mean those processes through which students gradually become acquainted with historically constituted cultural meanings and forms of reasoning and action. By subjectification we mean the processes through which the students take position in cultural practices and are shaped as culturally and historically unique subjects. Subjectification is the historical process of the creation of the unending creation of the self.

In a previous section we pointed out the role of consciousness in the definition of objectification and noted that consciousness should not be considered in metaphysical terms. Here, we consider consciousness from a dialectical-materialist viewpoint. Consciousness is the relationship between the individual and the cultural world. Or, as Vygotsky (1979, v. 17, n. 4, p. 31) stated in 1925: "consciousness must be seen as a particular case of the social experience". The structure of consciousness "is the relation [of the individual] with the external world" (VYGOTSKY, 1997, p. 137).

Leont'ev insisted on the idea that consciousness cannot be understood without understanding the individual's activity:

> man's (sic) consciousness . . . is not additive. It is not a flat surface, nor even a capacity that can be filled with images and processes. Nor is it the connections of its separate elements. It is the internal movement of its "formative elements" geared to the general movement of the activity which effects the *real* life of the individual in society. Man's activity is the substance of his consciousness. (LEONT'EV, 2009, p. 26).

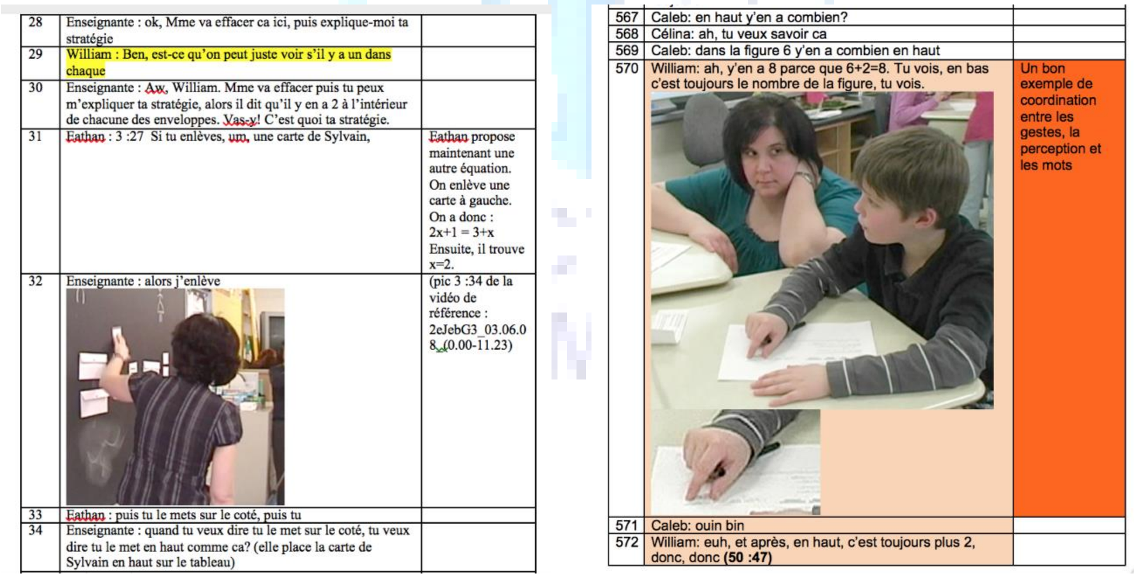
What Leont'ev is asserting is that consciousness is not something already there, a recipient to be filled with experience. Nor is consciousness the foundational basement of being (which was in fact Hegel's (1977) position and the position advocated by idealist philosophers in general). Consciousness is *movement*. The real concrete being, the real individual (the students, in our case) finds its foundational basement, the substance of her consciousness, in her *concrete activity*, that is, in her life. For, as Marx (2007, p. 121) asked in a profound passage written in a reading note, "What is life if not activity?"

Within the theory of objectification, consciousness has hence to be related to activity, which as was mentioned before, constitutes the theory's methodological unit of analysis. We track in the classroom activity those passages in which students become progressively aware of culturally constituted mathematical meanings. This awareness is empirically investigated, through the sensuous actions of the students, in perceptual, aural, kinesthetic, gestural, linguistic, and symbolic activity in general. This is why we track the students' and teachers' multimodal activity. Although we come from a logocentric tradition, that is, a tradition that emphasizes the role of language and discourse in knowing, we maintain that activity-based consciousness often emerges at a sensuous, pre-conceptual, and pre-intentional level (Radford, 2014c).

Within these methodological parameters, we start the data analysis with either a rough transcription of the videos or with a first analysis of the videos in order to select what we call "salient segments." Salient segments refer to passages that seem to contain the sought-after learning evidence. Once salient segments are identified, they are subjected to a transcription (if the transcription has not been done yet). This is followed by an *interpretative transcription analysis*, inspired by Fairclough (1995), Moerman (1988), and Coulthard (1977). The interpretative transcription analysis is carried out in three steps.

In the first step, all utterances are treated equally without paying attention to context, intention, and so on.

In the second step, the rough material resulting from the first step is analyzed through the lenses of the theoretical principles of the theory and the research questions at hand (see Figure 1). The salient segments (or parts of them) are identified and put into emerging conceptual analytical categories (e.g., types of gestures, symbol-production, symbol understanding) and then contextualized by adding: (1) pictures and the precise picture time in the video, and (2) interpretative comments that we insert in the third column of the transcription sheet (see Figure 7). The first and second columns have the transcription line number and the body of the transcription, respectively.



28	Enseignante : ok, Mme va effacer ca ici, puis explique-moi ta stratégie	
29	William : Ben, est-ce qu'on peut juste voir s'il y a un dans chaque	
30	Enseignante : Aw, William. Mme va effacer puis tu peux m'expliquer ta stratégie, alors il dit qu'il y en a 2 à l'intérieur de chacune des enveloppes. Vas-y! C'est quoi ta stratégie.	
31	Eathan : 3 :27 Si tu enlèves, um, une carte de Sylvain,	Eathan propose maintenant une autre équation. On enlève une carte à gauche. On a donc : 2x+1 = 3+x Ensuite, il trouve x=2.
32	Enseignante : alors j'enlève	(pic 3 :34 de la vidéo de référence : 2eJebG3_03.06.08_(0.00-11.23)
33	Eathan : puis tu le mets sur le coté, puis tu	
34	Enseignante : quand tu veux dire tu le met sur le coté, tu veux dire tu le met en haut comme ca? (elle place la carte de Sylvain en haut sur le tableau)	

567	Caleb: en haut y'en a combien?	
568	Célina: ah, tu veux savoir ca	
569	Caleb: dans la figure 6 y'en a combien en haut	
570	William: ah, y'en a 8 parce que 6+2=8. Tu vois, en bas c'est toujours le nombre de la figure, tu vois.	Un bon exemple de coordination entre les gestes, la perception et les mots
571	Caleb: ouin bin	
572	William: euh, et après, en haut, c'est toujours plus 2, donc, donc **(50 :47)**	

Figure 7- Two examples of data analysis. For the published results, see Radford (2014b) and Radford (2014d), respectively.

The second example in Figure 7 includes a picture of a student with the teacher a bit behind, and a close-up of the student's hand. The discussion is about the number of squares on the top row of Term 6 of the sequence shown in Figure 8. The close-up highlights the use of a meaningful gesture through which Grade 4 student William, talking to a teammate, Caleb, feels sensuously, so to speak, a mathematical relationship between variables that the year before was very difficult to conceive. The gesture is made up of two fingers that show the whole bottom row of Term 6 that 8-to-9-year-old William has drawn on his student activity sheet. Here, through a successful link of the verbal utterance and the gesture, the relationship between algebraic variables is manifested: the top row has 6+2 squares (see Figure 8).

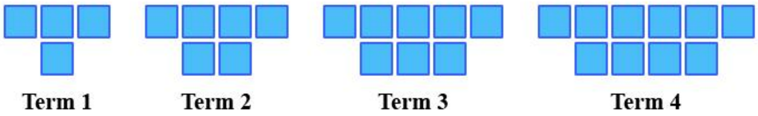


Figure 8 - The sequence that the Grade 4 students discuss.

In line 572, William refines the idea and says: “euh, on the top [row], it is always plus 2, so, so.” In line 653 (about six minutes later), Caleb states: “oui ok, double le nombre et ajoute 2” [Yes ok, double the number and add 2]. The way has now been paved to tackle the question of symbolizing the formula as 2xn+2 (see Radford, 2014d). Let me return to the three-step process of the interpretative transcript analysis.

In the third step, the cadence of the dialogue is inserted in the transcription by indicating pauses, verbal hesitations, the occurrence of gestures, etc. Depending on the granularity of the analysis we may also insert voice analyses with the help of *Praat*, a dedicated prosody analysis software (see Radford, Bardini, and Sabena, 2007; Radford and Sabena, 2015).

During the second and third steps, a frame-by-frame video analysis is conducted. Emerging conceptual analytical categories are further analyzed and refined with the help of NVivo software.

The resulting material serves as the potential highly processed data to be included in our publications and conference presentations. It might include edited videos of the original French videos (often subtitled in English, Spanish, Italian, Portuguese, or other languages) and edited pictures from videos, BS documents, scanned student activity sheets, and comments from field notes.

As we analyze our data, new developmental hypotheses are developed. They are taken into consideration in the design of new tasks, leading to the cycle shown in Figure 9.

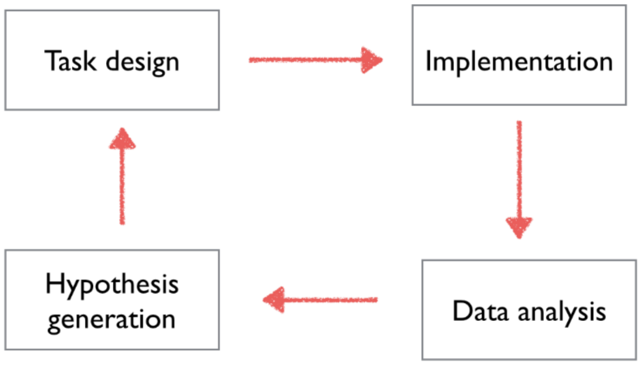


Figure 9 - The cycle of task design, implementation, data analysis, and hypothesis generation.

A question remains: in our analyses and the reports of our results, we are reporting phenomena pertaining to mathematics learning. Now, is it the learning of a student in particular or is it the learning of the group or even the class? In other words, are we reporting learning of a collective or of an individual?

Learning, we suggested, is related to consciousness. And individual consciousness, Vygotsky reminded us, is but a particular case of social consciousness. Leont'ev went a step forward and claimed that the substance of consciousness is *activity*. So, what we report is the learning of one, two, or a few individual students (that, usually although not necessarily, we take to be paradigmatic of the class). But (and this is the important point to take into consideration) we do not consider these students as isolated entities or monads. We consider them as *individuals-in-activit*y. In other words, we look at the *activity* for it is only through the prism of this unit of analysis that we can look conceptually to the students and come up with interpretations of the manners in which they are encountering cultural forms of thinking and action. When we fill progressively the second and third columns of our transcription sheet, it is not the student A or B or C that we look at, but student A, student B, student C, etc. *through the lenses of their joint activity* (to which we must add the teacher and the whole class). In progressively filling these columns, we try to add that which makes the activity a living phenomenon (pauses, hesitations, intonations, gestures, etc.). Still, in other words, we cannot investigate an activity without considering the specific and concrete individuals who are engaged in that activity. Activity *qua* activity is just a pure idealistic abstraction, much as is the idea of the Piagetian "epistemic subject" who lives in thin air as a phantasmagoric being. We are interested in the concrete student that breathes, suffers, and enjoys mathematics. We are not interested in abstractions. But at the same time we cannot see the breathing, suffering, enjoying student without considering the activity in which she engages and that constitutes the ultimate foundation of her being. To look at the individual *qua* individual is as much an unfortunate abstraction as the aforementioned opposite abstraction that looks at the activity or the collective without looking at the individuals who make the activity or the collective possible in first place.

Concluding remarks

In this article I have dealt with the question of the methodology of the theory of objectification. I started by arguing that a methodology is always linked to the theoretical principles and research questions of the theory (Figure 1). The methodology to which the theory presented here resorts attempts to track the processes of objectification and subjectification of which learning, it was argued, consists.

The unit of analysis is the *classroom activity*, which is conceptualized in a dialectic materialist sense. From a methodological viewpoint, the activity is not a homogeneous entity. It is not the activity of an isolated student or the activity of an isolated teacher, but an evolving individual-social phenomenon that moves towards an object (the object of the activity), even if such an object does not appear to each student with the same clarity and same understanding. The object of the activity is multifariously refracted and always changing in each one of the students' consciousness.

To produce learning evidence, we endeavour to track the manner in which the students come to encounter culturally constituted forms of thinking, imagining, intuiting, symbolizing, and acting. Premised by the idea that the texture of consciousness is of a semiotic nature and that "Semiotic analysis is the only adequate method for the study of the systemic and semantic structure of consciousness" (VYGOTSKY, 1997, p. 137), we track the multimodal semiotic dimension of students and teachers in classroom activity. Through a three-step *interpretative transcription analysis*, a multilayered reconstruction of the students' and teacher's discursive and non-discursive joint activity is thus created, allowing us to provide accounts of learning and those conditions that allow (or prevent) learning to occur.

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